



# Year 9

## PLOTTING AND INTERPRETTING GRAPHS

### Key Concept

**Substitution** – This is where you replace a number with a letter

If  $a = 5$  and  $b = 2$

$a + b =$	$5 + 2 = 7$
$a - b =$	$5 - 2 = 3$
$3a =$	$3 \times 5 = 15$
$ab =$	$5 \times 2 = 10$
$a^2 =$	$5^2 = 25$

### Key Words

**Intercept:** Where two graphs cross.

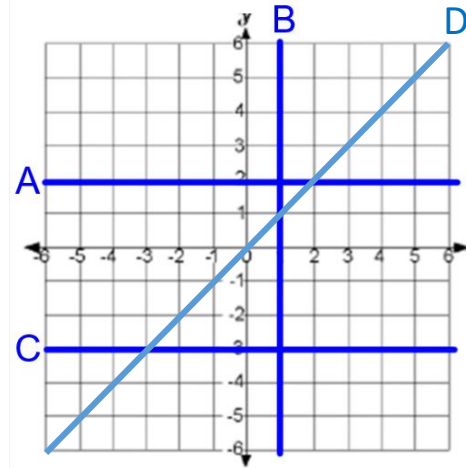
**Gradient:** This describes the steepness of the line.

**y-intercept:** Where the graph crosses the y-axis.

**Linear:** A linear graph is a straight line.

**Quadratic:** A quadratic graph is curved, u or n shape.

### Examples

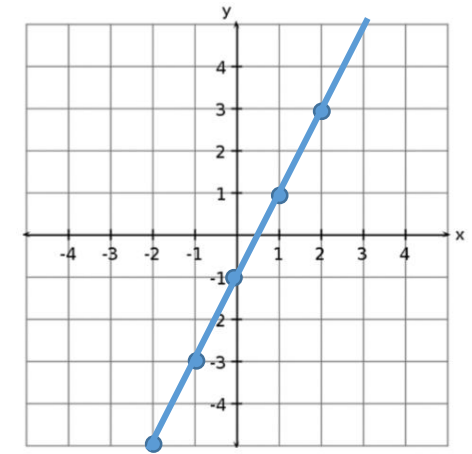


A:  $y = 2$     B:  $x = 1$

C:  $y = -3$     D:  $y = x$

Draw the graph of  $y = 2x - 1$

X	-2	-1	0	1	2
Y	-5	-3	-1	1	3



Notice this graph has a gradient of 2 and a y-intercept of -1.



Clip Numbers  
206 - 210, 251

### Tip

Parallel lines have the same gradient.

### Formula

$$\text{Gradient} = \frac{\text{difference in } y\text{'s}}{\text{difference in } x\text{'s}}$$

### Questions

1) What are the gradient and y-intercept of:

a)  $y = 4x - 3$

b)  $y = 4 + 6x$

c)  $y = -5x - 3$

2) Draw the graph of  $y = 3x - 2$  for x values from -3 to 3 using a table.

(c)  $m = -5, c = -3$

(b)  $m = 6, c = 4$

(a)  $m = 4, c = -3$

ANSWERS: 1) a)  $m = 4, c = -3$     b)  $m = 6, c = 4$     c)  $m = -5, c = -3$



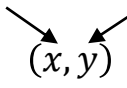
# Year 9

## STRAIGHT LINE GRAPHS AND EQUATION OF A LINE

### Key Concepts

**Coordinates** in 2D are written as follows:

$x$  is the value that is to the left/right  
 $y$  is the value that is to up/down



**Straight line graphs** always have the equation:

$$y = mx + c$$

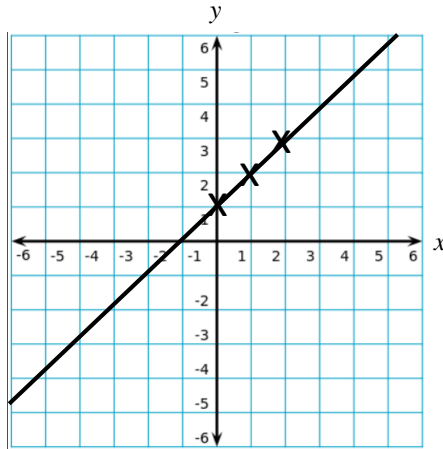
$m$  is the **gradient** i.e. the steepness of the graph.

$c$  is the **y intercept** i.e. where the graph cuts the  $y$  axis.

**Parallel** lines always have the same **gradient**.

Plot the graph of  $y = 2x + 1$

$x$	0	1	2
$y$	1	2	3



Examples of lines parallel to this graph are:  $y = 2x - 3$  or  $y = 2x + 7$

### Examples

Calculate the equation of this line:

$$y = mx + c$$

$$m = \frac{4}{2}$$

$$= 2$$

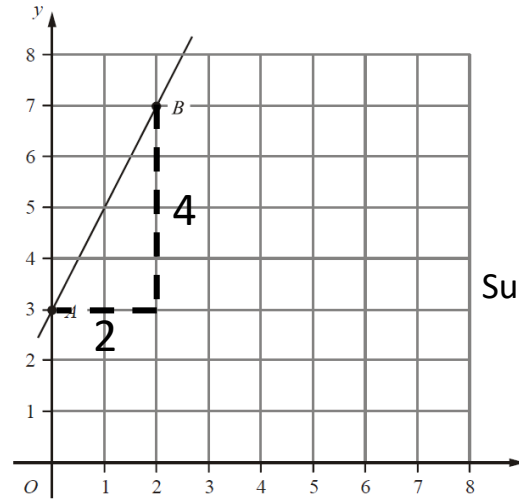
$$y = 2x + c$$

Substitute in a coordinate: (2,7)

$$7 = (2 \times 2) + c$$

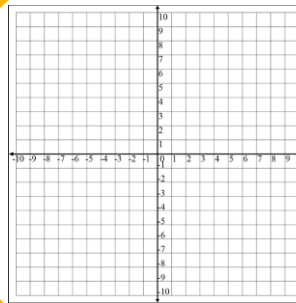
$$3 = c$$

$$y = 2x + 3$$

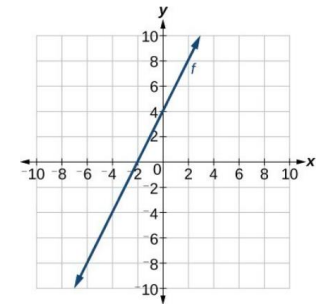


199,200,205,  
207-211,214

**Key Words**  
**Coordinate**  
**Gradient**  
**Parallel**



- 1) Plot the line  $y = 3x - 2$
- 2) Find the equation of the line for the attached graph.
- 3) State the equation of a line that would be parallel to this line.

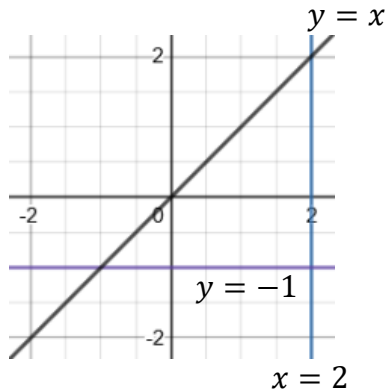




# Year 9

## COORDINATE GEOMETRY

### Key Concepts



**Parallel** lines have the same gradient.

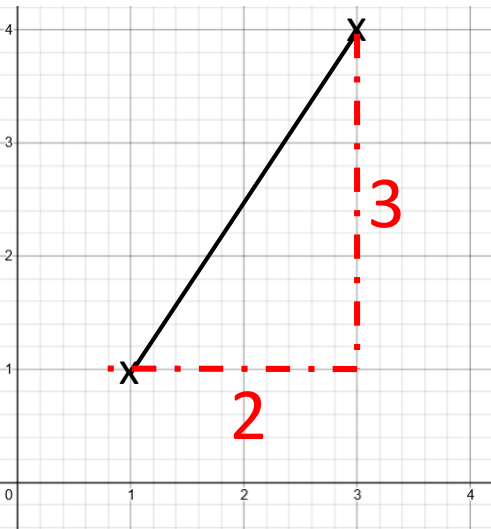
e.g.  $y = 2x + 3$  and  $y = 2x - 1$

**Perpendicular** line gradients are the negative reciprocal of one another

e.g.  $y = 2x$  and  $y = -\frac{1}{2}x$



200,  
205,  
208



### Key Words

Parallel  
Perpendicular  
Gradient  
Midpoint  
Length

### Examples

Calculate the midpoint between the coordinates (1,1) and (3,4).

$$\text{Midpoint} = \left( \frac{3 + 1}{2}, \frac{1 + 4}{2} \right) = (2, 2.5)$$

Calculate the distance between the coordinates (1,1) and (3,4).

$$\begin{aligned} \text{Length} &= \sqrt{2^2 + 3^2} \\ &= 3.61 \end{aligned}$$

- 1) Calculate the midpoint between the coordinates (2,7) and (9,11).
- 2) Calculate the distance between the coordinates (2,7) and (9,11).



# Year 9

## SIMULTANEOUS EQUATIONS

### Key Concepts

Simultaneous equations are when **more than one equation** are given, which involve **more than one variable**. The variables have the **same value** in each equation.

### Two linear equations:

$$\begin{array}{r} 3x + 2y = 18 \\ 3x - y = 9 \quad \times 2 \\ \hline \end{array}$$

$$\begin{array}{r} 3x + 2y = 18 \\ 6x - 2y = 18 \quad + \\ \hline 9x = 36 \\ x = 4 \end{array}$$

SSS – Same Sign Subtract  
DSA – Different Sign Add

Substitute in  $x = 4$  into an original equation

$$\begin{array}{r} 3x + 2y = 18 \\ (3 \times 4) + 2y = 18 \\ 12 + 2y = 18 \\ 2y = 6 \\ y = 3 \end{array}$$

### One linear and one quadratic equation: **Examples**

$$\begin{array}{r} x^2 + y^2 = 17 \\ y = x - 3 \end{array}$$

Substitute  $y = x - 3$  into  $y$  in the quadratic equation.

$$\begin{array}{r} x^2 + (x - 3)^2 = 17 \\ x^2 + x^2 - 6x + 9 - 17 = 0 \\ 2x^2 - 6x - 8 = 0 \end{array}$$

Solve by factorising or using the quadratic formula.

$$x = 4 \text{ or } x = -1$$

Substitute the  $x$  values into the linear equation to find the corresponding  $y$  values.

$$\text{When } x = 4, \quad y = 4 - 3 = 1$$

$$\text{When } x = -1, \quad y = -1 - 3 = -4$$



190-195

### Key Words

Simultaneous  
Substitution  
Elimination  
Linear  
Quadratic

Solve each set of simultaneous equations:

$$\begin{array}{l} 1) \quad 3x + 2y = 4 \\ \quad \quad 4x + 5y = 17 \end{array}$$

$$\begin{array}{l} 2) \quad x^2 + y^2 = 13 \\ \quad \quad x = y - 5 \end{array}$$

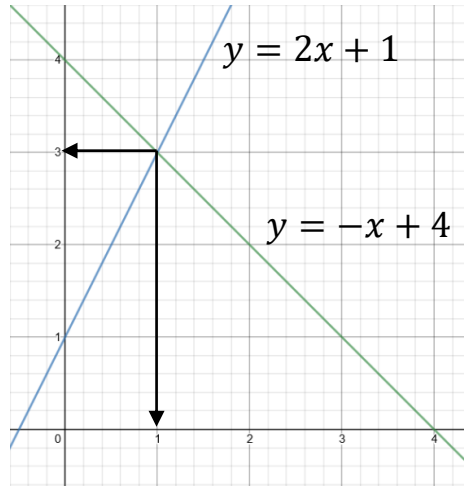
## SOLVE SIMULTANEOUS EQUATIONS GRAPHICALLY

### Key Concepts

Simultaneous equations are when **more than one equation** are given which involve **more than one variable**. The variables have the **same value** in each equation.

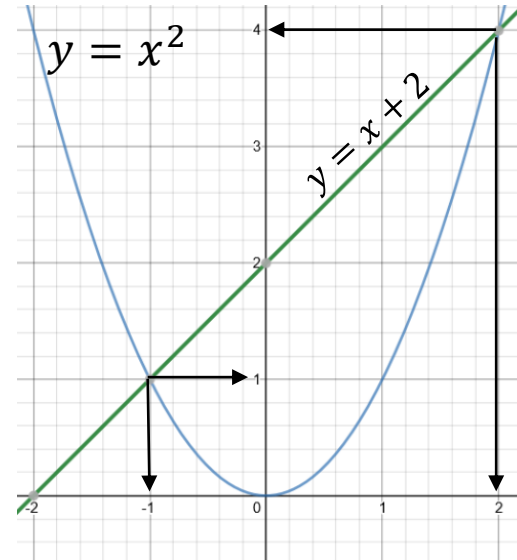
Simultaneous equations can be solved **graphically** whereby the **intersection** of the graphs gives the  $x$  and  $y$  values.

Solve graphically:  $y = 2x + 1$   
 $y = -x + 4$



$x = 1$  and  $y = 3$

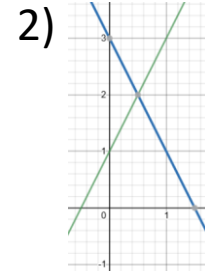
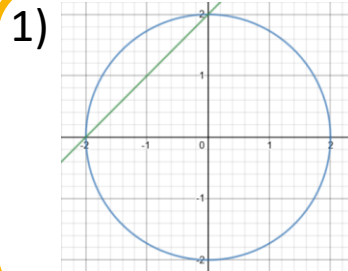
Solve graphically:  $y = x^2$   
 $y = x + 2$



$x = -1$  and  $y = 1$   
 $x = 2$  and  $y = 4$

### Examples

**Key Words**  
 Simultaneous  
 Equation  
 Intersection



Solve each set of simultaneous equations graphically.

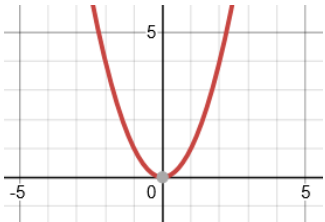


# Year 9 QUADRATIC GRAPHS

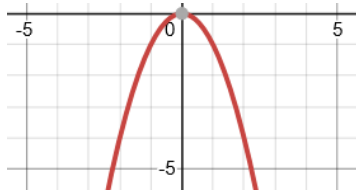
## Key Concepts

A quadratic graph will always be in the shape of a parabola.

$$y = x^2$$



$$y = -x^2$$



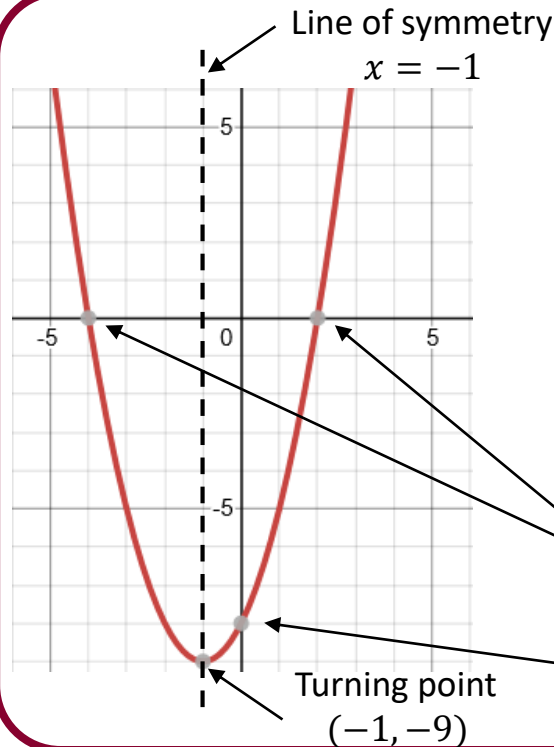
The roots of a quadratic graph are where the graph crosses the  $x$  axis. The roots are the solutions to the equation.

## Examples

$$y = x^2 + 2x - 8$$

A quadratic equation can be solved from its graph.

The roots of the graph tell us the possible solutions for the equation. There can be 1 root, 2 roots or no roots for a quadratic equation. This is dependant on how many times the graph crosses the  $x$  axis.



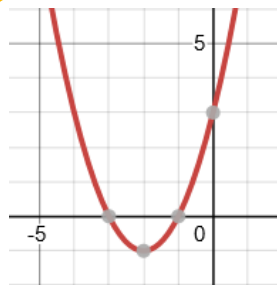
Roots  $x = -4$   
 $x = 2$

$y$  intercept =  $-8$

Turning point  
 $(-1, -9)$

## Key Words

Quadratic  
Roots  
Intercept  
Turning point  
Line of symmetry



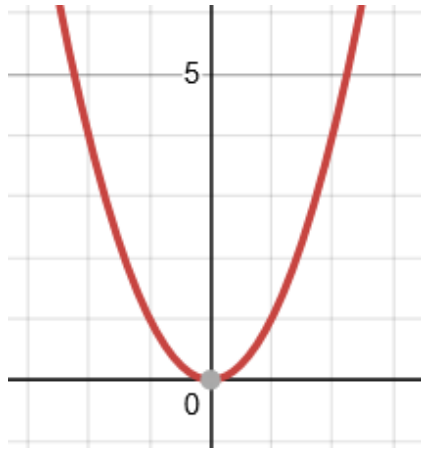
Identify from the graph of  $y = x^2 + 4x + 3$ :

- 1) The line of symmetry
- 2) The turning point
- 3) The  $y$  intercept
- 4) The two roots of the equation

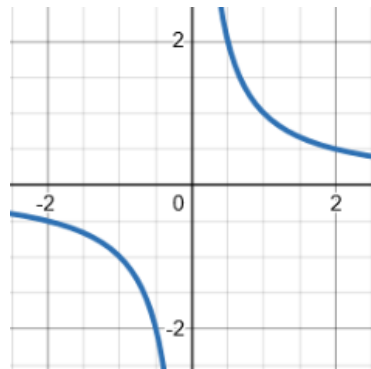
# Year 9

## TYPES OF GRAPH

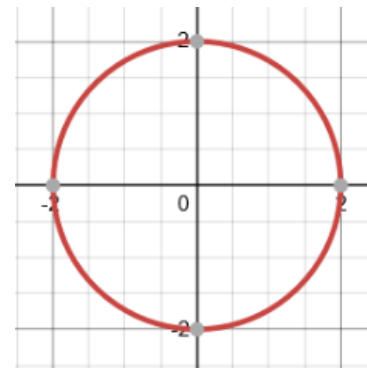
### Examples



Quadratic graphs  
 $y = x^2$



Reciprocal graphs  
 $y = \frac{1}{x}$



Circle graphs  
 $x^2 + y^2 = 4$

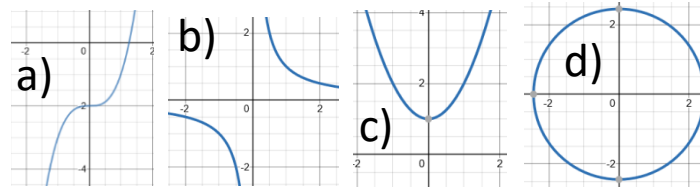


Cubic graphs  
 $y = x^3$

#### Key Words

Quadratic  
Cubic  
Reciprocal  
Circle  
Graph

Match the graph with the correct equation:

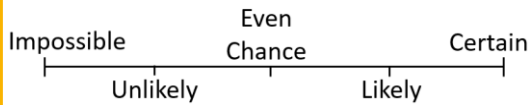


- 1)  $x^2 + y^2 = 6$
- 2)  $y = \frac{1}{x}$
- 3)  $y = x^3 - 2$
- 4)  $y = x^2 + 1$

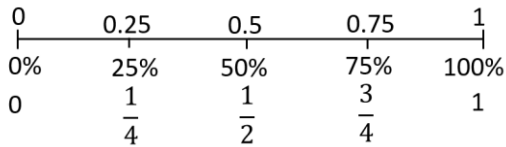
## INTRODUCING PROBABILITY

### Key Concept

#### Chance



#### Probability



Probabilities can be written as:

- Fractions
- Decimals
- Percentages

### Key Words

**Probability:** The chance of something happening as a numerical value.

**Impossible:** The outcome cannot happen.

**Certain:** The outcome will definitely happen.

**Even chance:** There are two different outcomes each with the same chance of happening.

**Expectation:** The amount of times you expect an outcome to happen based on probability.

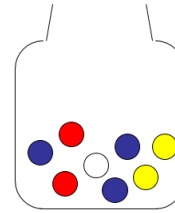
### Tip

Probabilities always add up to 1.

### Formula

*Expectation*  
= Probability  $\times$  no. of trials

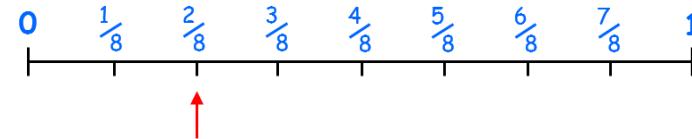
### Examples



- 1) What is the probability that a bead chosen will be **yellow**.  
Show the answer on a number line.

$$\text{Probability} = \frac{\text{Number of favourable outcomes}}{\text{Total number of outcomes}}$$

$$P(\text{Yellow}) = \frac{2}{8} = \frac{1}{4}$$



- 2) How many **yellow** beads would you **expect** if you pulled a bead out and replaced it 40 times?

$$\frac{1}{4} \times 40 = \frac{1}{4} \text{ of } 40 = 10$$

### Questions

In a bag of skittles there are 12 red, 9 yellow, 6 blue and 3 purple left.  
Find: a) P(Red) b) P(Yellow) c) P(Red or purple) d) P(Green)





# Year 9

## THEORETICAL PROBABILITY

### Key Concepts

**Probabilities** can be described using **words** and **numerically**.

We can use **fractions**, **decimals** or **percentages** to represent a probability.

**Theoretical probability** is what should happen if all variables were fair.

All probabilities must **add to 1**.

The probability of something **NOT** happening equals:

$$1 - (\text{probability of it happening})$$

### Probability scale:

Impossible	Even chance			Certain
$\frac{0}{4}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{4}{4}$
0	0.25	0.5	0.75	1
0%	25%	50%	75%	100%

### Examples

There are only red counters, blue counters, white counters and black counters in a bag.

Colour	Red	Blue	Black	White
No. of counters	9	3x	x-5	2x

There are only red counters, blue counters, white counters and black counters in a bag.

Colour	Red	Blue	Black	White
No. of counters	9	3	5	2

A counter is chosen at random, the probability it is red is  $\frac{9}{100}$ . Work out the probability it is black.

$$9 + 3x + x - 5 + 2x = 100$$

$$6x + 4 = 100$$

$$x = 16$$

$$\text{Number of black counters} = 16 - 5 = 11$$

$$\text{Probability of choosing black} = \frac{11}{100}$$

- What is the probability that a blue counter is chosen?  $\frac{3}{19} = \frac{\text{number of blue}}{\text{total number of counters}}$
- What is the probability that red is **not** chosen?  $\frac{10}{19} = \frac{\text{number of all other colours}}{\text{total number of counters}}$



## 349-353

**Key Words**  
**Theoretical Probability**  
**Fraction**  
**Decimal**  
**Percentage**  
**Certain**  
**Impossible**  
**Even chance**

	1	2	3
Prob	5	4	9

	1	2	3
Prob	0.37	2x	x

- Calculate the probability of choosing a 2.
- Calculate the probability of not choosing a 3.



# Year 9

## RELATIVE FREQUENCY

### Key Concepts

**Experimental probability** differs to theoretical probability in that it is based upon the **outcomes from experiments**. It may not reflect the outcomes we expect.

Experimental probability is also known as the **relative frequency** of an event occurring.

**Estimating** the number of times an event will occur:

$$\text{Probability} \times \text{no. of trials}$$



355-357

**Key Words**  
Experimental  
Relative  
frequency  
Fraction  
Decimal  
Probability  
Estimate

### Examples

Colour	red	blue	white	black
Prob	$x$	0.2	0.3	$x$

A spinner is spun, it has four colours on it.  
The relative frequencies of each colour are recorded.  
The relative frequency of red and black are the same.

a) What is the relative frequency of red?

$$1 - (0.2 + 0.3) = 0.5$$
$$x = \frac{0.5}{2} = 0.25$$

b) If the spinner is spun 300 times, how many times do you expect it to land on white?

$$0.3 \times 300 = 90$$

Number	1	2	3	4
Prob	$x$	0.46	0.28	$x$

A spinner is spun which has 1,2,3,4 on it. The probability that a 1 and a 4 are spun are equal.

a) What is the probability that a 4 is landed on?

b) If the spinner is spun 500 times how many times do we expect it to land on a 2?



# Year 9

## LISTING OUTCOMES AND SAMPLE SPACE

### Key Concepts

When there are a number of different possible outcomes in a situation we need a **logical** and **systematic** way in which to view them all.

We can be asked to **list** all possible outcomes e.g. choices from a menu, order in which people finish a race.

We can also use a **sample space diagram**. This records the possible outcomes of two different events happening.



358-359,  
370-371

**Key Words**  
List  
Outcome  
Sample space  
Probability

### Examples

Starter	Main
Fishcake Melon	Lasagne Beef Salmon

List all of the combinations possible when one starter and one main are chosen.

F, L    M, L  
F, B    M, B  
F, S    M, S

Note: You can write the initials of each option in a test. You do not need to write out the full word.

Two dice are thrown and the possible outcomes are shown in the sample space diagram below:

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

- 1) What is the probability that 2 numbers which are the same are rolled?

$$\frac{6}{36} = \frac{\text{outcomes where numbers are the same}}{\text{total number of outcomes}}$$

- 2) What is the probability that two even numbers are rolled?

$$\frac{9}{36} = \frac{\text{outcomes where numbers are both even}}{\text{total number of outcomes}}$$

- 1) Abe, Ben and Carl have a race. List all of the options for the order that the boys can end the race.

		Spinner		
		Red	Green	Blue
Coin	Heads	H,R	H,G	H,B
	Tails	T,R	T,G	T,B

- 2a) What is the probability that a head is landed on?  
b) What is the probability that a head and a green are landed on?



# Year 9

## TWO WAY TABLES AND PROBABILITY TABLES

### Key Concepts

**Two way tables** are used to tabulate a number of pieces of information.

Probabilities can be formulated easily from two way tables.

**Probabilities** can be written as a **fraction, decimal or a percentage** however we often work with fractions. You do not need to simplify your fractions in probabilities.

**Estimating** the number of times an event will occur

$$\text{Probability} \times \text{no. of trials}$$



353,  
422-424

### Examples

There are only red counters, blue counters, white counters and black counters in a bag.

Colour	Red	Blue	Black	White
No. of counters	9	3x	x-5	2x

A counter is chosen at random, the probability it is red is  $\frac{9}{100}$ . Work out the probability it is black.

$$9 + 3x + x - 5 + 2x = 100$$

$$6x + 4 = 100$$

$$x = 16$$

$$\text{Number of black counters} = 16 - 5$$

$$= 11$$

$$\text{Probability of choosing black} = \frac{11}{100}$$

80 children went on a school trip. They went to London or to York.  
23 boys and 19 girls went to London. 14 boys went to York.

	London	York	Total
Girls	19	24	43
Boys	23	14	37
Total	42	38	80

What is the probability that a person is chosen that went to London?  $\frac{42}{80}$

If a girl is chosen, what is the probability that she went to York?  $\frac{24}{38}$

**Key Words**  
Two way table  
Probability  
Fraction  
Outcomes  
Frequency

	1	2	3
Prob	0.37	2x	x

- 1a) Calculate the probability of choosing a 2 or a 3.  
b) Estimate the number of times a 2 will be chosen if the experiment is repeated 300 times.

2a) Complete the two way table:

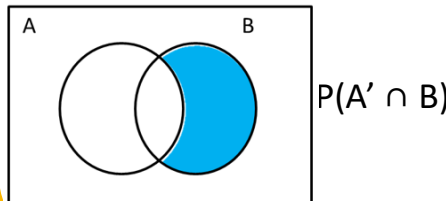
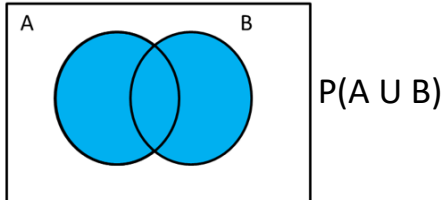
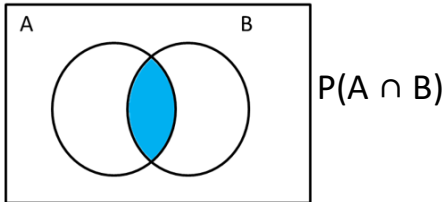
	Year Group			Total
	9	10	11	
Boys			125	407
Girls		123		
Total	303	256		831

b) What is the probability that a Y10 is chosen, given that they are a girl .

# Year 9

## FURTHER PROBABILITY

### Key Concept



### Key Words

**Probability:** The chance of something happening as a numerical value.

**Impossible:** The outcome cannot happen.

**Certain:** The outcome will definitely happen.

**Even chance:** There are two different outcomes each with the same chance of happening.

**Mutually Exclusive:** Two events that cannot both occur at the same time.

### Formula

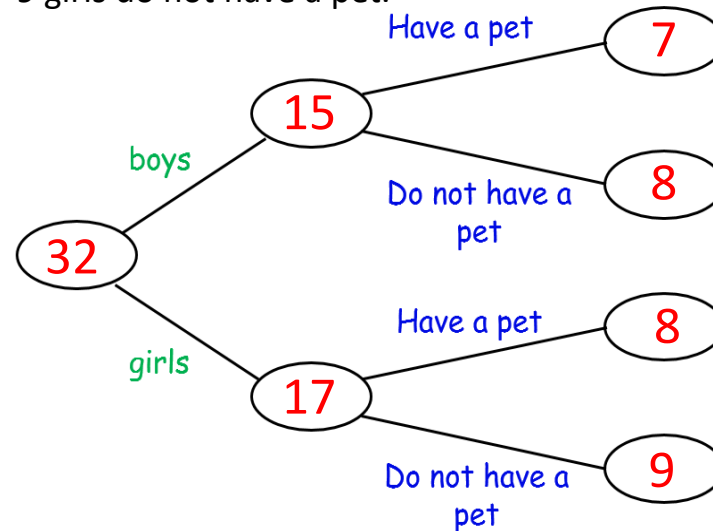
$$P(A \cap B) = P(A) \times P(B)$$

$$P(A \cup B) = P(A) + P(B)$$

or (non ME)  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

### Examples

In Hannah's class there are 32 students.  
15 of these students are boys.  
7 of the boys have a pet.  
9 girls do not have a pet.



$$P(\text{boy}) = \frac{15}{32}$$

$$P(\text{Girl with pet}) = \frac{8}{32}$$

### Questions

- 1) Draw a two-way table for the question above.
- 2) Find the probability that a pupil chosen is a boy with no pets.
- 3) A girl is chosen, what is the probability she has a pet?



# Year 9

## PROBABILITY TREE DIAGRAMS

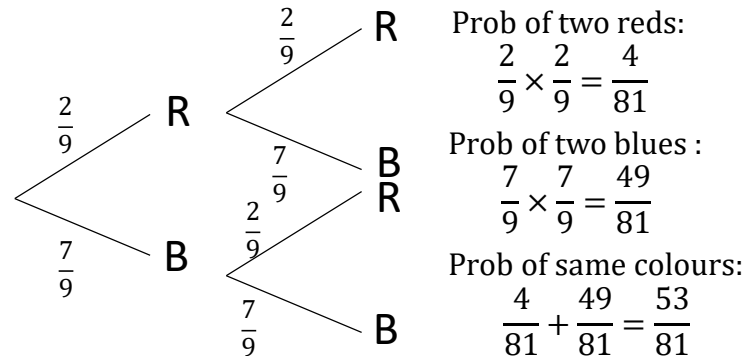
### Key Concepts

**Independent events** are events which do not affect one another.

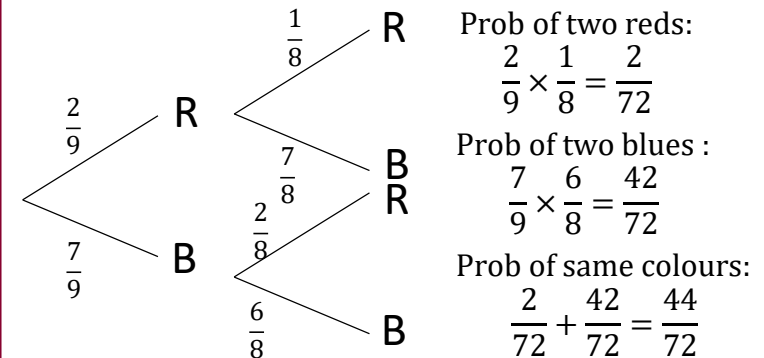
**Dependent events** affect one another's probabilities. This is also known as **conditional probability**.

### Examples

There are red and blue counters in a bag.  
 The probability that a red counter is chosen is  $\frac{2}{9}$ .  
 A counter is chosen and **replaced**, then a second counter is chosen.  
 Draw a tree diagram and calculate the probability that two counters of the same colour are chosen.



There are red and blue counters in a bag.  
 The probability that a red counter is chosen is  $\frac{2}{9}$ .  
 A counter is chosen and **not replaced**, then a second counter is chosen.  
 Draw a tree diagram and calculate the probability that two counters of the same colour are chosen.



361-362,  
 364-367,  
 389-390

**Key Words**  
 Independent  
 Dependant  
 Conditional  
 Probability  
 Fraction

1) There are blue and green pens in a drawer. There are 4 blues and 7 greens. A pen is chosen and then **replaced**, then a second pen is chosen. Draw a tree diagram to show this information and calculate the probability that pens of different colours are chosen.

2) There are blue and green pens in a drawer. There are 4 blues and 7 greens. A pen is chosen and **not replaced**, then a second pen is chosen. Draw a tree diagram to show this information and calculate the probability that pens of different colours are chosen.



# Year 9

## VENN DIAGRAMS

### Key Concepts

Venn diagrams show all possible relationships between different sets of data.

Probabilities can be derived from Venn diagrams. Specific notation is used for this:

$P(A \cap B)$  = Probability of A **and** B

$P(A \cup B)$  = Probability of A **or** B

$P(A')$  = Probability of **not** A

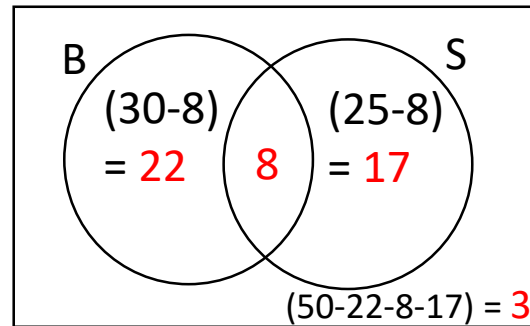
### Example

Out of 50 people surveyed:

30 have a brother

25 have a sister

8 have both a brother and sister



a) Complete the Venn diagram

b) Calculate:

i)  $P(A \cap B) = \frac{8}{50}$     ii)  $P(A \cup B) = \frac{47}{50}$     iii)  $P(B') = \frac{20}{50}$

iv) The probability that a person with a sister, does not have a brother.

$$= \frac{8}{25}$$

40 students were surveyed:

20 have visited France

15 have visited Spain

10 have visited both France and Spain

a) Complete a Venn diagram to represent this information.

b) Calculate:

i)  $P(F \cap S)$     ii)  $P(F \cup S)$     iii)  $P(S')$

iv) The probability someone who has visited France, has not gone to Spain.